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Reconstructing the chargino system at e^+e^- linear colliders

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Abstract. In most supersymmetric theories charginos, $\tilde{\chi}^{\pm}_{1,2}$, belong to the class of the lightest supersymmetric particles. The chargino system can be reconstructed completely in e^+e^- collider experiments: $e^+e^- \to \tilde{\chi}^+_i \tilde{\chi}^-_j$ $[i, j = 1, 2]$. By measuring the total cross sections and the asymmetries with polarized beams, the chargino masses and the gaugino–higgsino mixing angles of these states can be determined accurately. If only the lightest charginos $\tilde{\chi}^{\pm}_1$ are kinematically accessible in a first phase of the machine, transverse beam polarization or the measurement of chargino polarization in the final state is needed to determine the mixing angles. From these observables the fundamental SUSY parameters can be derived: the SU(2) gaugino mass M_2, the modulus and the cosine of the CP–violating phase of the higgsino mass parameter μ, and $\tan\beta = v_2/v_1$, the ratio of the vacuum expectation values of the two neutral Higgs doublet fields. The remaining two–fold ambiguity of the phase can be resolved by measuring the normal polarization of the charginos. Sum rules of the cross sections can be exploited to investigate the closure of the two–chargino system.

1 Introduction

In supersymmetric theories, the spin-1/2 partners of the $W^\pm$ gauge bosons and the charged Higgs bosons, $\tilde{W}^\pm$ and $\tilde{H}^\pm$, mix to form chargino mass eigenstates $\tilde{\chi}^\pm_{1,2}$. The chargino mass matrix [1] in the $(\tilde{W}^-, \tilde{H}^-)$ basis

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W\cos\beta \\ \sqrt{2}m_W\sin\beta & \mu \end{pmatrix} \tag{1}$$

is built up by the fundamental supersymmetry (SUSY) parameters: the SU(2) gaugino mass M_2, the higgsino mass parameter μ, and the ratio $\tan\beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields which break the electroweak symmetry. In CP–noninvariant theories, the mass parameters are complex [1]. However, by reparametrization of the fields, M_2 can be assumed real and positive without loss of generality so that the only non–trivial reparametrization–invariant phase may be attributed to μ:

$$\mu = |\mu|\,\mathrm{e}^{i\Phi_\mu} \quad \text{with} \quad 0 \le \Phi_\mu \le 2\pi \tag{2}$$

Once charginos will have been discovered, the experimental analysis of their properties in production and decay mechanisms will reveal the basic structure of the underlying supersymmetric theory.

Charginos are produced in e^+e^- collisions, either in diagonal or in mixed pairs [2]-[11]:

$$e^+e^- \;\to\; \tilde{\chi}^+_i\,\tilde{\chi}^-_j \quad [\,i, j = 1, 2\,]$$

Depending on the collider energy and the chargino masses, the following scenarios will be analyzed:

(i) If the energy in the first phase of the machine is only sufficient to produce the light chargino pair $\tilde{\chi}^+_1\tilde{\chi}^-_1$, the underlying fundamental parameters, up to at most two–fold ambiguity, can be extracted from the mass $m_{\tilde{\chi}^\pm_1}$, the total production cross section and the measurement of longitudinal left–right and transverse asymmetries. Alternatively to beam polarization, the polarization of the charginos in the final state may be exploited. The $\tilde{\chi}^\pm$ polarization vectors and $\tilde{\chi}^+$–$\tilde{\chi}^-$ spin–spin correlation tensor can be determined from the decay distributions of the charginos. We will assume that the charginos decay into the lightest neutralino $\tilde{\chi}^0_1$, which is taken to be stable, and a pair of quarks and antiquarks or leptons: $\tilde{\chi}^\pm_1 \to \tilde{\chi}^0_1 f\bar{f}'$. No detailed information on the decay dynamics, nor on the structure of the neutralino, is needed to carry out the spin analysis [12].
(ii) If the collider energy is sufficient to produce the two chargino states in pairs, the underlying fundamental SUSY parameters $\{M_2, |\mu|, \cos\Phi_\mu, \tan\beta\}$ can be extracted *unambiguously* from the masses $m_{\tilde{\chi}^\pm_{1,2}}$, the total production

cross sections, and the left–right (LR) asymmetries with polarized electron beams, while the phase Φ_μ is determined up to a two–fold ambiguity $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$. As shown in [13], this ambiguity can be resolved by measuring manifestly CP–noninvariant observables associated with the normal polarization of the charginos.

These analyses of the chargino sector are independent of the structure of the neutralino sector [14]. While the structure of the chargino sector in large classes of supersymmetric theories is isomorphic to the minimal supersymmetric standard model (MSSM), we expect the neutralino sector to be more complex in general, reflecting the complexity of a Higgs sector extended beyond the minimal form.

The analysis will be based strictly on low–energy SUSY. To clarify the analytical structure, the reconstruction of the basic SUSY parameters presented here is carried out at the tree level; the small loop corrections [15] include parameters from other sectors of the MSSM demanding iterative higher–order expansions in global analyses at the very end. Once these basic parameters will have been extracted experimentally, they may be confronted, for instance, with the ensemble of relations predicted in Grand Unified Theories.

In this report we present a coherent and comprehensive description of the chargino system at e^+e^- linear colliders, based on scattered elements discussed earlier in [5]–[7]. The report will be divided into six parts. In Sect. 2 we recapitulate the central elements of the mixing formalism for the charged gauginos and higgsinos. In Sect. 3 the cross sections for chargino production, the left–right asymmetries, and the polarization vectors of the charginos are given. In Sect. 4 we describe a phenomenological analysis of the light $\tilde{\chi}_1^\pm$ states based on a specific scenario to exemplify the procedure for extracting the fundamental SUSY parameters in a model–independent way. In Sect. 5 the analysis is extended to the complete set $\tilde{\chi}_{1,2}^\pm$ of chargino states, leading to an unambiguous determination of the SU(2) gaugino parameters. Conclusions are given in Sect. 6.

2 Mixing formalism

In the MSSM and many of its extensions, the two charginos $\tilde{\chi}_{1,2}^\pm$ are mixtures of the charged SU(2) gauginos and higgsinos. As a consequence of possible field redefinitions, the parameters $\tan\beta$ and M_2 can be chosen real and positive. Since the chargino mass matrix $\mathcal{M}_C$ is not symmetric, two different unitary matrices acting on the left– and right–chiral $(\tilde{W}, \tilde{H})_{L,R}$ two–component states

$$U_{L,R} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}^- \end{pmatrix}_{L,R} = \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix}_{L,R} \tag{3}$$

are needed to diagonalize the matrix (1). The unitary matrices U_L and U_R can be parameterized in the following way [13]:

$$\begin{aligned} U_L &= \begin{pmatrix} \cos\phi_L & \mathrm{e}^{-i\beta_L}\sin\phi_L \\ -\mathrm{e}^{i\beta_L}\sin\phi_L & \cos\phi_L \end{pmatrix} \\ U_R &= \begin{pmatrix} \mathrm{e}^{i\gamma_1} & 0 \\ 0 & \mathrm{e}^{i\gamma_2} \end{pmatrix} \begin{pmatrix} \cos\phi_R & \mathrm{e}^{-i\beta_R}\sin\phi_R \\ -\mathrm{e}^{i\beta_R}\sin\phi_R & \cos\phi_R \end{pmatrix} \end{aligned} \tag{4}$$

The mass eigenvalues $m^2_{\tilde{\chi}^\pm_{1,2}}$ are given by

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}\left[M_2^2 + |\mu|^2 + 2m_W^2 \mp \Delta_C\right] \tag{5}$$

with Δ_C involving the phase Φ_μ:

$$\begin{aligned} \Delta_C = &\big[(M_2^2 - |\mu|^2)^2 + 4m_W^4\cos^2 2\beta + 4m_W^2(M_2^2 + |\mu|^2) \\ &+ 8m_W^2 M_2|\mu|\sin 2\beta\cos\Phi_\mu\big]^{1/2} \end{aligned} \tag{6}$$

The quantity Δ_C determines the difference of the two chargino masses: $\Delta_C = m^2_{\tilde{\chi}_2^\pm} - m^2_{\tilde{\chi}_1^\pm}$. The four phase angles $\{\beta_L, \beta_R, \gamma_1, \gamma_2\}$ are not independent but can be expressed in terms of the invariant angle Φ_μ:

$$\begin{aligned} \tan\beta_L &= -\frac{\sin\Phi_\mu}{\cos\Phi_\mu + \frac{M_2}{|\mu|}\cot\beta} \\ \tan\beta_R &= +\frac{\sin\Phi_\mu}{\cos\Phi_\mu + \frac{M_2}{|\mu|}\tan\beta} \\ \tan\gamma_1 &= +\frac{\sin\Phi_\mu}{\cos\Phi_\mu + \frac{M_2\left[m^2(\tilde{\chi}_1^\pm) - |\mu|^2\right]}{|\mu| m_W^2 \sin 2\beta}} \\ \tan\gamma_2 &= -\frac{\sin\Phi_\mu}{\cos\Phi_\mu + \frac{M_2 m_W^2 \sin 2\beta}{|\mu|\left[m^2(\tilde{\chi}_2^\pm) - M_2^2\right]}} \end{aligned} \tag{7}$$

All four phase angles vanish in CP–invariant theories for which $\Phi_\mu = 0$ or π. The rotation angles ϕ_L and ϕ_R satisfy the relations:

$$\begin{aligned} \cos 2\phi_{L,R} &= -\left[M_2^2 - |\mu|^2 \mp 2m_W^2\cos 2\beta\right]/\Delta_C \\ \sin 2\phi_{L,R} &= -2m_W\left[M_2^2 + |\mu|^2 \pm (M_2^2 - |\mu|^2)\cos 2\beta\right. \\ &\quad \left. + 2M_2|\mu|\sin 2\beta\cos\Phi_\mu\right]^{1/2}/\Delta_C \end{aligned} \tag{8}$$

The two rotation angles $\phi_{L,R}$ and the phase angles $\{\beta_L, \beta_R, \gamma_1, \gamma_2\}$ define the couplings of the chargino–chargino–Z vertices:

$$\begin{aligned} \langle\tilde{\chi}^-_{1L}|Z|\tilde{\chi}^-_{1L}\rangle &= -\frac{g_W}{c_W}\left[s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_L\right] \\ \langle\tilde{\chi}^-_{1R}|Z|\tilde{\chi}^-_{1R}\rangle &= -\frac{g_W}{c_W}\left[s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_R\right] \\ \langle\tilde{\chi}^-_{1L}|Z|\tilde{\chi}^-_{2L}\rangle &= +\frac{g_W}{4c_W}\,\mathrm{e}^{-i\beta_L}\sin 2\phi_L \\ \langle\tilde{\chi}^-_{1R}|Z|\tilde{\chi}^-_{2R}\rangle &= +\frac{g_W}{4c_W}\,\mathrm{e}^{-i(\beta_R - \gamma_1 + \gamma_2)}\sin 2\phi_R \\ \langle\tilde{\chi}^-_{2L}|Z|\tilde{\chi}^-_{2L}\rangle &= -\frac{g_W}{c_W}\left[s_W^2 - \frac{3}{4} + \frac{1}{4}\cos 2\phi_L\right] \\ \langle\tilde{\chi}^-_{2R}|Z|\tilde{\chi}^-_{2R}\rangle &= -\frac{g_W}{c_W}\left[s_W^2 - \frac{3}{4} + \frac{1}{4}\cos 2\phi_R\right] \end{aligned}$$

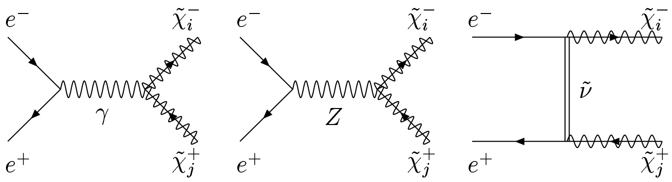

Fig. 1. The three mechanisms contributing to the production of chargino pairs $\tilde{\chi}_i^- \tilde{\chi}_j^+$ in e^+e^- collisions

and the electron–sneutrino–chargino vertices:

$$\begin{aligned} \langle \tilde{\chi}_{1R}^- | \tilde{\nu} | e_L^- \rangle &= -g_{\tilde{W}} \, \mathrm{e}^{i\gamma_1} \cos\phi_R \\ \langle \tilde{\chi}_{2R}^- | \tilde{\nu} | e_L^- \rangle &= +g_{\tilde{W}} \, \mathrm{e}^{i(\beta_R+\gamma_2)} \sin\phi_R \end{aligned} \tag{9}$$

with $s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W$ denoting the electroweak mixing angle. g_W and $g_{\tilde{W}}$ are the $e\nu W$ gauge coupling and the $e\tilde{\nu}\tilde{W}$ Yukawa coupling, respectively. They are identical in supersymmetric theories:

$$g_{\tilde{W}} = g_W = e/s_W \tag{10}$$

Since the coupling to the higgsino component, which is proportional to the electron mass, can be neglected in the sneutrino vertex, the sneutrino couples only to left–handed electrons. The diagonal and L/R symmetric photon–chargino vertices are as usual

$$\langle \tilde{\chi}_i^- | \gamma | \tilde{\chi}_i^- \rangle = e \tag{11}$$

CP–violating effects are manifest only in mixed $\tilde{\chi}_1\tilde{\chi}_2$ pairs.

Conversely, the fundamental SUSY parameters M_2, $|\mu|$, $\tan\beta$ and the phase parameter $\cos\Phi_\mu$ can be extracted from the chargino $\tilde{\chi}_{1,2}^\pm$ parameters: the masses $m_{\tilde{\chi}_{1,2}^\pm}$ and the two mixing angles ϕ_L and ϕ_R of the left– and right–chiral components of the wave function (see Sect. 5).

3 Chargino production in e^+e^- collisions

The production of chargino pairs at e^+e^- colliders is based on three mechanisms: s–channel γ and Z exchanges, and t–channel $\tilde{\nu}_e$ exchange, cf. Fig. 1. The transition matrix element, after a Fierz transformation of the $\tilde{\nu}_e$–exchange amplitude,

$$\begin{aligned} T[e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+] = \frac{e^2}{s} Q_{\alpha\beta} \left[\bar{v}(e^+) \gamma_\mu P_\alpha u(e^-) \right] \\ \times \left[\bar{u}(\tilde{\chi}_i^-) \gamma^\mu P_\beta v(\tilde{\chi}_j^+) \right] \end{aligned} \tag{12}$$

can be expressed in terms of four bilinear charges, defined by the chiralities $\alpha, \beta = L, R$ of the associated lepton and chargino currents. After introducing the following notation,

$$\begin{aligned} D_L &= 1 + \frac{D_Z}{s_W^2 c_W^2} \left(s_W^2 - \frac{1}{2} \right) \left(s_W^2 - \frac{3}{4} \right) \\ F_L &= \frac{D_Z}{4 s_W^2 c_W^2} \left(s_W^2 - \frac{1}{2} \right) \\ D_R &= 1 + \frac{D_Z}{c_W^2} \left(s_W^2 - \frac{3}{4} \right) \\ F_R &= \frac{D_Z}{4 c_W^2} \end{aligned} \tag{13}$$

and

$$\begin{aligned} D'_L &= D_L + \left(\frac{g_{\tilde{W}}}{g_W} \right)^2 \frac{D_{\tilde{\nu}}}{4 s_W^2} \\ F'_L &= F_L - \left(\frac{g_{\tilde{W}}}{g_W} \right)^2 \frac{D_{\tilde{\nu}}}{4 s_W^2} \end{aligned} \tag{14}$$

the four bilinear charges $Q_{\alpha\beta}$ are linear in the mixing parameters $\cos 2\phi_{L,R}$ and $\sin 2\phi_{L,R}$; for the diagonal $\tilde{\chi}_1^- \tilde{\chi}_1^+$, $\tilde{\chi}_2^- \tilde{\chi}_2^+$ modes and the mixed mode $\tilde{\chi}_1^- \tilde{\chi}_2^+$ we find:

$$\begin{aligned} \{11\}/\{22\}: \quad Q_{LL} &= D_L \mp F_L \cos 2\phi_L \\ Q_{RL} &= D_R \mp F_R \cos 2\phi_L \\ Q_{LR} &= D'_L \mp F'_L \cos 2\phi_R \\ Q_{RR} &= D_R \mp F_R \cos 2\phi_R \end{aligned} \tag{15}$$

$$\begin{aligned} \{12\}/\{21\}: \quad Q_{LL} &= F_L \, \mathrm{e}^{\mp i\beta_L} \sin 2\phi_L \\ Q_{RL} &= F_R \, \mathrm{e}^{\mp i\beta_L} \sin 2\phi_L \\ Q_{LR} &= F'_L \, \mathrm{e}^{\mp i(\beta_R - \gamma_1 + \gamma_2)} \sin 2\phi_R \\ Q_{RR} &= F_R \, \mathrm{e}^{\mp i(\beta_R - \gamma_1 + \gamma_2)} \sin 2\phi_R \end{aligned} \tag{16}$$

The first index in $Q_{\alpha\beta}$ refers to the chirality of the $e^\pm$ current, the second index to the chirality of the $\tilde{\chi}^\pm$ current. The $\tilde{\nu}$ exchange affects only the LR chirality charge Q_{LR} while all other amplitudes are built up by γ and/or Z exchanges only. The first term in $D_{L,R}$ is generated by the γ exchange; $D_Z = s/(s - m_Z^2 + i m_Z \Gamma_Z)$ denotes the Z propagator and $D_{\tilde{\nu}} = s/(t - m_{\tilde{\nu}}^2)$ the $\tilde{\nu}$ propagator with momentum transfer t. The non–zero Z width can in general be neglected for the energies considered in the present analysis so that the charges are rendered complex in the Born approximation only through the CP–noninvariant phase.

For the sake of convenience we introduce eight quartic charges for each of the production processes of the diagonal and mixed chargino pairs, respectively. These charges [16] correspond to independent helicity amplitudes which describe the chargino production processes for polarized electrons/positrons with negligible lepton masses. Expressed in terms of bilinear charges they are collected in Table 1, including the transformation properties under P and CP.

The charges Q_1 to Q_5 are manifestly parity–even, Q'_1 to Q'_3 are parity–odd. The charges Q_1 to Q_3, Q_5, and Q'_1 to Q'_3 are CP–invariant[1]. Q_4 changes sign under CP transformations[2], yet depends only on one combination

[1] When expressed in terms of the fundamental SUSY parameters, these charges do depend nevertheless on $\cos\Phi_\mu$ indirectly through $\cos 2\phi_{L,R}$, in the same way as the $\tilde{\chi}_{1,2}^\pm$ masses depend indirectly on this parameter.

[2] The P–odd and CP–even/CP–odd counterparts to Q_5/Q_4, which carry a negative sign between the corresponding L and R components, do not affect the observables under consideration.

Table 1. The independent quartic charges of the chargino system, the measurement of which determines the chargino mass matrix

P	CP	Quartic charges
even	even	$Q_1 = \frac{1}{4}\left[\lvert Q_{RR}\rvert^2 + \lvert Q_{LL}\rvert^2 + \lvert Q_{RL}\rvert^2 + \lvert Q_{LR}\rvert^2\right]$
		$Q_2 = \frac{1}{2}\mathrm{Re}\left[Q_{RR}Q_{RL}^* + Q_{LL}Q_{LR}^*\right]$
		$Q_3 = \frac{1}{4}\left[\lvert Q_{RR}\rvert^2 + \lvert Q_{LL}\rvert^2 - \lvert Q_{RL}\rvert^2 - \lvert Q_{LR}\rvert^2\right]$
		$Q_5 = \frac{1}{2}\mathrm{Re}\left[Q_{LR}Q_{RR}^* + Q_{LL}Q_{RL}^*\right]$
	odd	$Q_4 = \frac{1}{2}\mathrm{Im}\left[Q_{RR}Q_{RL}^* + Q_{LL}Q_{LR}^*\right]$
odd	even	$Q_1' = \frac{1}{4}\left[\lvert Q_{RR}\rvert^2 + \lvert Q_{RL}\rvert^2 - \lvert Q_{LR}\rvert^2 - \lvert Q_{LL}\rvert^2\right]$
		$Q_2' = \frac{1}{2}\mathrm{Re}\left[Q_{RR}Q_{RL}^* - Q_{LL}Q_{LR}^*\right]$
		$Q_3' = \frac{1}{4}\left[\lvert Q_{RR}\rvert^2 + \lvert Q_{LR}\rvert^2 - \lvert Q_{RL}\rvert^2 - \lvert Q_{LL}\rvert^2\right]$

$(\beta_L - \beta_R + \gamma_1 - \gamma_2)$ of the CP angles. The CP invariance of Q_2 and Q_2' can easily be proved by noting that

$$\begin{aligned}&2m_{\tilde{\chi}_1^\pm} m_{\tilde{\chi}_2^\pm}\cos(\beta_L - \beta_R + \gamma_1 - \gamma_2)\sin 2\phi_L \sin 2\phi_R \\ &\quad = (m^2_{\tilde{\chi}_1^\pm} + m^2_{\tilde{\chi}_2^\pm})\,(1 - \cos 2\phi_L \cos 2\phi_R) - 4m_W^2 \quad (17)\end{aligned}$$

Therefore, all the production cross sections $\sigma[e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-]$ for any combination of pairs $\tilde{\chi}_i^+\tilde{\chi}_j^-$ depend only on $\cos 2\phi_L$ and $\cos 2\phi_R$ apart from the chargino masses, the sneutrino mass and the Yukawa couplings. For longitudinally–polarized electron beams, the sums and differences of the quartic charges are restricted to either L or R components (first index) of the $e^\pm$ currents.

Defining the $\tilde{\chi}_i^-$ production angle with respect to the electron flight–direction by the polar angle Θ and the azimuthal angle Φ with respect to the electron transverse polarization, the helicity amplitudes can be derived from (12). While electron and positron helicities are opposite to each other in all amplitudes, the $\tilde{\chi}_i^-$ and $\tilde{\chi}_j^+$ helicities are in general not correlated due to the non–zero masses of the particles; amplitudes with equal $\tilde{\chi}_i^-$ and $\tilde{\chi}_j^+$ helicities are reduced only to order $\propto m_{\tilde{\chi}_{i,j}^\pm}/\sqrt{s}$ for asymptotic energies. The helicity amplitudes may be expressed as $T_{ij}(\sigma;\lambda_i,\lambda_j) = 2\pi\alpha\, \mathrm{e}^{i\sigma\Phi}\langle\sigma;\lambda_i\lambda_j\rangle$, denoting the electron helicity by the first index σ, the $\tilde{\chi}_i^-$ and $\tilde{\chi}_j^+$ helicities by the remaining two indices, λ_i and λ_j, respectively. The explicit form of the helicity amplitudes $\langle\sigma;\lambda_i\lambda_j\rangle$ can be found in [6].

3.1 Production cross sections

Since the gaugino and higgsino interactions depend on the chirality of the states, the polarized electron and positron beams are powerful tools to reveal the composition of charginos. To describe the electron and positron polarizations, the reference frame must be fixed. The electron–momentum direction will define the z–axis and the electron transverse polarization–vector the x–axis. The azimuthal angle of the transverse polarization–vector of the positron is called η with respect to the x–axis. In this notation, the polarized differential cross section is given in terms of the electron and positron polarization vectors $P{=}(P_T, 0, P_L)$ and $\bar{P}{=}(\bar{P}_T\cos\eta, \bar{P}_T\sin\eta, -\bar{P}_L)$ by

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{16s}\lambda^{1/2}\Big[&(1 - P_L\bar{P}_L)\,\Sigma_{\mathrm{unp}} + (P_L - \bar{P}_L)\,\Sigma_{LL} \\ &+ P_T\bar{P}_T\cos(2\Phi - \eta)\,\Sigma_{TT}\Big] \qquad (18)\end{aligned}$$

with the coefficients Σ_{unp}, Σ_{LL}, Σ_{TT} depending only on the polar angle Θ, but not on the azimuthal angle Φ any more; $\lambda = [1-(\mu_i+\mu_j)^2][1-(\mu_i-\mu_j)^2]$ is the two–body phase space function, and $\mu_i^2 = m^2_{\tilde{\chi}_i^\pm}/s$. The coefficients Σ_{unp}, Σ_{LL}, and Σ_{TT} can be expressed in terms of the quartic charges:

$$\begin{aligned}\Sigma_{\mathrm{unp}} &= 4\Big\{\left[1 - (\mu_i^2 - \mu_j^2)^2 + \lambda\cos^2\Theta\right]Q_1 \\ &\qquad + 4\mu_i\mu_j Q_2 + 2\lambda^{1/2}Q_3\cos\Theta\Big\} \\ \Sigma_{LL} &= 4\Big\{\left[1 - (\mu_i^2 - \mu_j^2)^2 + \lambda\cos^2\Theta\right]Q_1' \\ &\qquad + 4\mu_i\mu_j Q_2' + 2\lambda^{1/2}Q_3'\cos\Theta\Big\} \\ \Sigma_{TT} &= -4\lambda\sin^2\Theta\, Q_5 \qquad (19)\end{aligned}$$

If the production angles could be measured unambiguously on an event–by–event basis, the quartic charges could be extracted directly from the angular dependence of the cross section at a single energy. However, since charginos decay into the invisible lightest neutralinos and SM fermion pairs, the production angles cannot be determined completely on an event–by–event basis. The transverse distribution can be extracted by using an appropriate weight function for the azimuthal angle Φ. This leads us to the following integrated polarization–dependent cross sections as physical observables:

$$\begin{aligned}\sigma_R &= \int \mathrm{d}\Omega\, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\left[P_L = -\bar{P}_L = +1\right] \\ \sigma_L &= \int \mathrm{d}\Omega\, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\left[P_L = -\bar{P}_L = -1\right] \\ \sigma_T &= \int \mathrm{d}\Omega\, \left(\frac{\cos 2\Phi}{\pi}\right)\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\left[P_T = \bar{P}_T = 1;\ \eta = \pi\right] \qquad (20)\end{aligned}$$

As a result, nine independent physical observables can be constructed at a given c.m. energy by means of beam polarization in the three production processes; three in each mode $\{ij\} = \{11\}, \{12\}$ and $\{22\}$.

3.2 Chargino polarization and spin correlations

If the lepton beams are not polarized, the chiral structure of the charginos can be inferred from the polarization of the $\tilde{\chi}_i^-\tilde{\chi}_j^+$ pairs produced in e^+e^- annihilation.

The polarization vector $\vec{\mathcal{P}} = (\mathcal{P}_T, \mathcal{P}_N, \mathcal{P}_L)$ is defined in the rest frame of the particle $\tilde{\chi}_i^-$. $\mathcal{P}_L$ denotes the component parallel to the $\tilde{\chi}_i^-$ flight direction in the c.m. frame, $\mathcal{P}_T$ the transverse component in the production plane, and $\mathcal{P}_N$ the component normal to the production plane. The longitudinal and transverse components of the $\tilde{\chi}_i^-$ polarization vector can easily be expressed in terms of the quartic charges:

$$\begin{aligned}
\mathcal{P}_L &= 4\left\{2(1-\mu_i^2-\mu_j^2)\,\cos\Theta\, Q'_1 + 4\mu_i\mu_j\,\cos\Theta\, Q'_2\right.\\
&\quad \left.+\lambda^{1/2}[1+\cos^2\Theta-(\mu_i^2-\mu_j^2)]\,Q'_3\right\}/\mathcal{N}\\
\mathcal{P}_T &= -8\sin\Theta\left\{[(1-\mu_i^2+\mu_j^2)\,Q'_1+\lambda^{1/2}\,Q'_3\cos\Theta]\mu_i\right.\\
&\quad \left.+(1+\mu_i^2-\mu_j^2)\mu_j\,Q'_2\right\}/\mathcal{N}\\
\mathcal{P}_N &= 8\lambda^{1/2}\mu_j\,\sin\Theta\, Q_4/\mathcal{N} \qquad (21)
\end{aligned}$$

with the normalization $\mathcal{N}$ given by

$$\begin{aligned}
\mathcal{N} = 4\left\{\left[1-(\mu_i^2-\mu_j^2)^2+\lambda\cos^2\Theta\right]Q_1\right.\\
\left.+4\mu_i\mu_j Q_2 + 2\lambda^{1/2} Q_3\cos\Theta\right\} \qquad (22)
\end{aligned}$$

The normal component $\mathcal{P}_N$ can only be generated by complex production amplitudes. Non-zero phases are present in the fundamental supersymmetric parameters if CP is broken in the supersymmetric interaction [1]. Also, the non–zero width of the Z boson and loop corrections generate non–trivial phases; however, the width effect is negligible for high energies and the effects due to radiative corrections are small. Neglecting loops and the small Z–width, the normal $\tilde{\chi}_1^-$ and $\tilde{\chi}_1^+$ polarizations in $e^+e^- \to \tilde{\chi}_1^-\tilde{\chi}_1^+$ are zero since the $\tilde{\chi}_1\tilde{\chi}_1\gamma$ and $\tilde{\chi}_1\tilde{\chi}_1 Z$ vertices are real even for non-zero phases in the chargino mass matrix, and the sneutrino–exchange amplitude is real, too. The same holds true for $\tilde{\chi}_2^-\tilde{\chi}_2^+$ production. Only for non-diagonal $\tilde{\chi}_1^-\tilde{\chi}_2^+/\tilde{\chi}_2^-\tilde{\chi}_1^+$ pairs the amplitudes are complex giving rise to a non–zero CP–violating normal chargino polarization $\mathcal{P}_N$ with

$$\begin{aligned}
\mathcal{P}_N[\tilde{\chi}_{1,2}^-] = \pm 4\lambda^{1/2}\mu_{2,1}\left(F_R^2 - F_L F_L'\right)\,\sin\Theta\,\sin 2\phi_L\\
\times\sin 2\phi_R\,\sin(\beta_L-\beta_R+\gamma_1-\gamma_2)/\mathcal{N} \quad (23)
\end{aligned}$$

Below, we will concentrate on the production of the lightest charginos. The direct measurement of chargino polarization would provide detailed information on the three quartic charges Q_1', Q_2', Q_3'. However, the polarization of charginos can only be determined indirectly from angular distribution of decay products provided the chargino decay dynamics is known. Complementary information can be obtained from the observation of spin–spin correlations. Since they are reflected in the angular correlations between the $\tilde{\chi}_1^-$ and $\tilde{\chi}_1^+$ decay products, some of them are experimentally accessible directly. Moreover, taking suitable combinations of polarization and spin–spin correlations, any dependence on the specific parameters of the chargino decay mechanisms can be eliminated.

The polarization and spin–spin correlations of the charginos are encoded in the angular distributions of the decay products. Assuming the neutralino $\tilde{\chi}_1^0$ to be the lightest supersymmetric particle, several mechanisms contribute to the decay of the chargino $\tilde{\chi}_1^\pm$:

$$\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 + f\bar{f}' \qquad [f, f' = l, \nu, q]$$

Choosing the $\tilde{\chi}_1^\pm$ flight direction as quantization axis, the polar angles of the $f\bar{f}'$ decay systems in the $\tilde{\chi}_1^-/\tilde{\chi}_1^+$ rest frames are defined as θ^* and $\bar{\theta}^*$, respectively, and the corresponding azimuthal angles with respect to the production plane by ϕ^* and $\bar{\phi}^*$. The spin analysis–powers κ and $\bar{\kappa}$ are the coefficients of those parts of the $\tilde{\chi}_1^-$ and $\tilde{\chi}_1^+$ spin–density matrices which are different from the unit matrix. The κ's are built up by the decay form factors. In many scenarios typical numerical values of κ's are of the order of a few 10^{-1}. For the subsequent analysis they need not be determined in detail; it is enough to verify experimentally that they are sufficiently large.

Integrating out the unobserved production angle Θ of the charginos and the invariant masses of the final–state quark or leptonic systems $f_i\bar{f}_j$, the differential distribution can be written in terms of sixteen independent angular parts:

$$\begin{aligned}
&\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*\mathrm{d}\phi^*\mathrm{d}\cos\bar{\theta}^*\mathrm{d}\bar{\phi}^*} \sim \varSigma_{\rm unpol}\\
&+\cos\theta^*\kappa\,\mathcal{P} + \cos\bar{\theta}^*\bar{\kappa}\,\bar{\mathcal{P}}\\
&+\cos\theta^*\cos\bar{\theta}^*\kappa\bar{\kappa}\,\mathcal{Q}\\
&+\sin\theta^*\sin\bar{\theta}^*\cos(\phi^*+\bar{\phi}^*)\kappa\bar{\kappa}\,\mathcal{Y} + \ldots \qquad (24)
\end{aligned}$$

$\varSigma_{\rm unpol}$ is the integrated cross section summed over chargino polarizations; it can be expressed in terms of the quartic charges Q_1, Q_2, Q_3 in analogy to (19):

$$\begin{aligned}
\varSigma_{\rm unpol} = 4\int \mathrm{d}\cos\Theta\ \left\{(1+\beta^2\cos^2\Theta)\,Q_1\right.\\
\left.+(1-\beta^2)\,Q_2 + 2\beta\,\cos\Theta\, Q_3\right\} \qquad (25)
\end{aligned}$$

where $\beta = \sqrt{1-4m^2_{\tilde{\chi}_1^\pm}/s}$ is the $\tilde{\chi}_1^\pm$ velocity in the c.m. frame. Among the polarization vectors, only the integrated longitudinal components are useful in the present context, being proportional to

$$\begin{aligned}
\mathcal{P} = 4\int \mathrm{d}\cos\Theta\ \left\{(1+\beta^2)\,\cos\Theta\, Q'_1\right.\\
\left.+4(1-\beta^2)\,\cos\Theta\, Q'_2 + (1+\cos^2\Theta)\beta\, Q'_3\right\} \ (26)
\end{aligned}$$

for $\tilde{\chi}_1^-$ and $\bar{\mathcal{P}}$ for $\tilde{\chi}_1^+$ correspondingly. The spin correlation $\mathcal{Q}$ measures the difference between the cross sections for like–sign and unlike–sign $\tilde{\chi}_1^-$ and $\tilde{\chi}_1^+$ helicities; $\mathcal{Y}$ measures the interference between the amplitudes for positive and negative helicities of both the charginos. They can be expressed in terms of the quartic charges Q_1 to Q_3 as

$$\begin{aligned}
\mathcal{Q} &= -4\int \mathrm{d}\cos\Theta\,\left[(\beta^2+\cos^2\Theta)\,Q_1\right.\\
&\quad \left.+(1-\beta^2)\cos^2\Theta\, Q_2 + 2\beta\cos\Theta\, Q_3\right]\\
\mathcal{Y} &= -2\int \mathrm{d}\cos\Theta(1-\beta^2)\,[Q_1+Q_2]\sin^2\Theta \qquad (27)
\end{aligned}$$

The terms introduced explicitly in (24) are particularly interesting as they can be measured directly in terms of laboratory observables as follows.

The decay angles $\{\theta^*, \phi^*\}$ and $\{\bar\theta^*, \bar\phi^*\}$, which are used to measure the $\tilde\chi_1^\pm$ chiralities, are defined in the rest frame of the charginos $\tilde\chi_1^-$ and $\tilde\chi_1^+$, respectively. Since two invisible neutralinos are present in the final state, they cannot be reconstructed completely. However, the longitudinal components and the inner product of the transverse components can be reconstructed[3] from the momenta and energies measured in the laboratory frame (see e.g. [17,5]),

$$\begin{aligned}
\cos\theta^* &= \frac{1}{\beta|\vec{p^*}|}\left(\frac{E}{\gamma} - E^*\right), \\
\cos\bar\theta^* &= \frac{1}{\beta|\vec{\bar{p}^*}|}\left(\frac{\bar E}{\gamma} - \bar E^*\right) \\
\sin\theta^*\sin\bar\theta^*\cos(\phi^*+\bar\phi^*) &= \frac{|\vec p||\vec{\bar p}|}{|\vec{p^*}||\vec{\bar{p}^*}|}\cos\vartheta \\
&+ \frac{(E - E^*/\gamma)\left(\bar E - \bar E^*/\gamma\right)}{\beta^2|\vec{p^*}||\vec{\bar{p}^*}|}
\end{aligned} \tag{28}$$

where $\gamma = \sqrt{s}/2m_{\tilde\chi_1^\pm}$. $E(\bar E)$ and $E^*(\bar E^*)$ are the energies of the two hadronic systems in the $\tilde\chi_1^-$ and $\tilde\chi_1^+$ decays, defined in the laboratory frame and in the rest frame of the charginos, respectively; $\vec p(\vec{\bar p})$ and $\vec p^*(\vec{\bar p}^*)$ are the corresponding momenta. ϑ is the angle between the momenta of the two hadronic systems in the laboratory frame; the angle between the vectors in the transverse plane is given by $\Delta\phi^* = 2\pi - (\phi^* + \bar\phi^*)$ for the reference frames defined earlier. The terms in (24) can therefore be measured directly. The observables $\mathcal{P}$, $\bar{\mathcal{P}}$, $\mathcal{Q}$ and $\mathcal{Y}$ enter into the cross section together with the spin analysis-power $\kappa(\bar\kappa)$. CP–invariance leads to the relation $\bar\kappa = -\kappa$. Therefore, taking the ratios $\mathcal{P}\bar{\mathcal{P}}/\mathcal{Q}$ and $\mathcal{P}\bar{\mathcal{P}}/\mathcal{Y}$, these unknown quantities can be eliminated so that the two ratios reflect unambiguously the properties of the chargino system, not affected by the neutralinos. It is thus possible to study the chargino sector in isolation by measuring the mass of the lightest chargino, the total production cross section and the spin(–spin) correlations.

Since the polarization $\mathcal{P}$ is odd under parity and charge–conjugation, it is necessary to identify the chargino electric charges in this case. This can be accomplished by making use of the mixed leptonic and hadronic decays of the chargino pairs. On the other hand, the observables $\mathcal{Q}$ and $\mathcal{Y}$ are defined without charge identification so that the dominant hadronic decay modes of the charginos can be exploited.

4 Masses, mixing angles and couplings

Before the strategies for measuring the masses, mixing angles and the couplings are presented in detail, a few general remarks on the structure of the chargino system may render the techniques more transparent.

(i) The right–handed cross sections σ_R do not involve the exchange of the sneutrino. They depend only, in symmetric form, on the mixing parameters $\cos 2\phi_L$ and $\cos 2\phi_R$.
(ii) The left–handed cross sections σ_L and the transverse cross section σ_T depend on $\cos 2\phi_{L,R}$, the sneutrino mass and the $e\tilde\nu\tilde W$ Yukawa coupling. Thus the sneutrino mass and the Yukawa coupling can be determined from the left-handed and transverse cross sections. [If the sneutrino mass is much larger than the collider energy, only the ratio of the Yukawa coupling over the sneutrino mass squared $(g^2_{\tilde W}/m^2_{\tilde\nu})$ can be measured by this method [18].]

The cross sections σ_L, σ_R and σ_T are binomials in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane. If the two–chargino model is realized in nature, any two contours, σ_L and σ_R for example, will at least cross at one point in the plane between $-1 \le \cos 2\phi_L, \cos 2\phi_R \le +1$. However, the contours, being ellipses or hyperbolae, may cross up to four times. This ambiguity can be resolved by measuring the third physical quantity, σ_T for example. The measurement of σ_T is particularly important if the sneutrino mass is unknown. While the curve for σ_R is fixed, the curve for σ_L will move in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane with changing $m_{\tilde\nu}$. However, the third curve will intersect the other two in the same point only if the mixing angles as well as the sneutrino mass correspond to the correct physical values.

The numerical analyses presented below have been worked out for the two parameter points introduced in [19]. They correspond to a small and a large $\tan\beta$ solution for universal gaugino and scalar masses at the GUT scale:

$$\begin{aligned}
&\boldsymbol{RR1}: (\tan\beta, m_0, M_{\frac12}) = (\ 3, 100\,\mathrm{GeV}, 200\,\mathrm{GeV}) \\
&\boldsymbol{RR2}: (\tan\beta, m_0, M_{\frac12}) = (30, 160\,\mathrm{GeV}, 200\,\mathrm{GeV})
\end{aligned} \tag{29}$$

The CP-phase Φ_μ is set to zero. The induced chargino $\tilde\chi^\pm_{1,2}$, neutralino $\tilde\chi^0_1$ and sneutrino $\tilde\nu$ masses are given as follows:

$$\begin{aligned}
m_{\tilde\chi_1^\pm} &= 128/132\,\mathrm{GeV} & m_{\tilde\chi_1^0} &= 70/72\,\mathrm{GeV} \\
m_{\tilde\chi_2^\pm} &= 346/295\,\mathrm{GeV} & m_{\tilde\nu} &= 166/206\,\mathrm{GeV}
\end{aligned} \tag{30}$$

for the two points $\boldsymbol{RR1/2}$, respectively. The size of the unpolarized total cross sections $\sigma[e^+e^- \to \tilde\chi_i^+\tilde\chi_j^-]$ as functions of the collider energy is shown for two reference points in Fig. 2. With the maximum of the cross sections in the range of 0.1 to 0.3 pb, about 10^5 to 3×10^5 events can be generated for an integrated luminosity $\int\mathcal{L} \simeq 1$ ab^{-1} as planned in three years of running at TESLA.

The cross sections for chargino pair–production rise steeply at the threshold,

$$\sigma[e^+e^- \to \tilde\chi_i^+\tilde\chi_j^-] \sim \sqrt{s - (m_{\tilde\chi_i^\pm} + m_{\tilde\chi_j^\pm})^2} \tag{31}$$

so that the masses $m_{\tilde\chi_1^\pm}$, $m_{\tilde\chi_2^\pm}$ can be measured very accurately in the production processes of the final–state pairs

[3] The neutralino mass which enters this analysis, can be predetermined in a model–independent way from the endpoints of the chargino decay spectra.

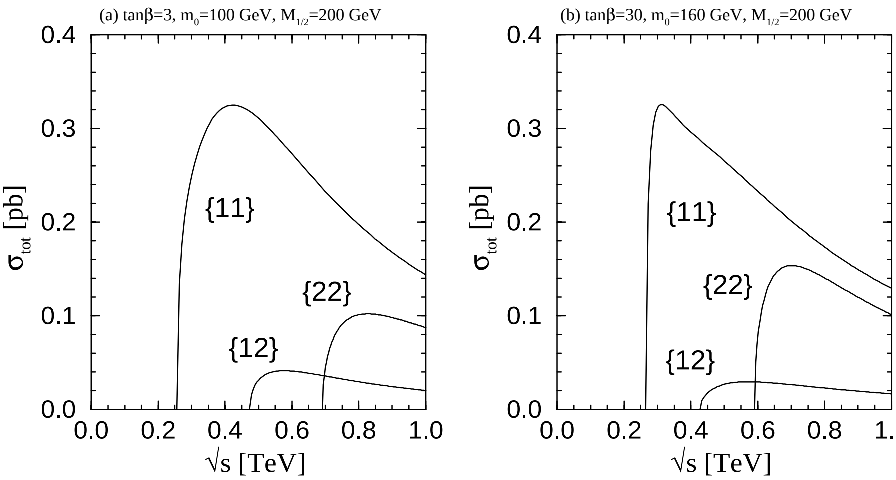

Fig. 2. The cross sections for the production of charginos as a function of the c.m. energy **a** with the ***RR*1** set and **b** with the ***RR*2** set of the fundamental SUSY parameters

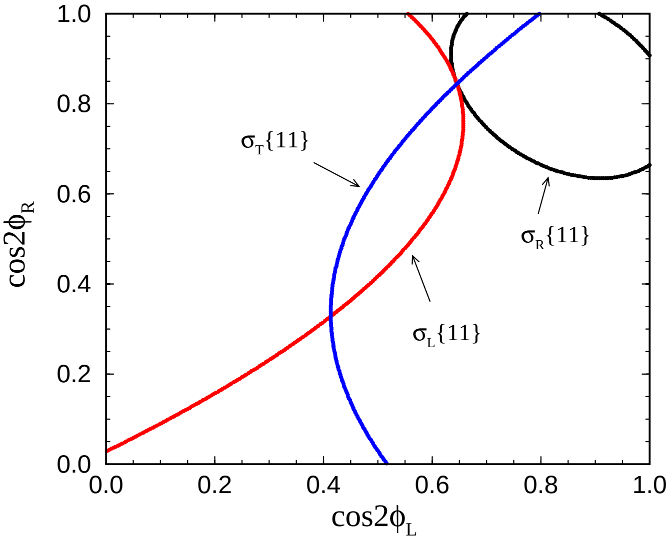

Fig. 3. Contours of the cross sections $\sigma_L\{11\}$, $\sigma_R\{11\}$ and $\sigma_T\{11\}$ in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane for the set ***RR*1** [$\tan\beta = 3$, $m_0 = 100$ GeV, $M_{1/2} = 200$ GeV] at the e^+e^- c.m. energy of 400 GeV

$\{11\}$, $\{12\}$ and $\{22\}$. Detailed experimental simulations have shown that accuracies $\Delta m_{\tilde{\chi}_1^\pm} = 40$ MeV and $\Delta m_{\tilde{\chi}_2^\pm} = 250$ MeV can be achieved in high–luminosity threshold scans [20].

4.1 Light chargino pair production

At an early phase of the e^+e^- linear collider the energy may only be sufficient to reach the threshold of the light chargino pair $\tilde{\chi}_1^+\tilde{\chi}_1^-$. Nearly the entire structure of the chargino system can nevertheless be reconstructed even in this case.

4.1.1 Exploiting longitudinal and transverse beam polarization

By analyzing the $\{11\}$ mode in $\sigma_L\{11\}$, $\sigma_R\{11\}$, the mixing angles $\cos 2\phi_L$ and $\cos 2\phi_R$ can be determined up to at most a four–fold ambiguity if the sneutrino mass is known and the Yukawa coupling is identified with the gauge coupling. The ambiguity can be resolved by adding the information from $\sigma_T\{11\}$. This is demonstrated[4] in Fig. 3 for the reference point ***RR*1** at the energy $\sqrt{s} = 400$ GeV. Moreover, the additional measurement of the transverse cross section can be exploited to determine the sneutrino mass. While the right–handed cross section σ_R does not depend on $m_{\tilde{\nu}_e}$, the contours σ_L, σ_T move uncorrelated in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane until the correct sneutrino mass is used in the analysis. The three contour lines intersect exactly in one point of the plane only if all the parameters correspond to the correct physical values.

4.1.2 Chargino polarization

Without longitudinal and transverse beam polarizations, the polarization of the charginos in the final state and their spin–spin correlations can be used to determine the mixing angles $\cos 2\phi_L$ and $\cos 2\phi_R$.

The observables $\mathcal{P}$, $\bar{\mathcal{P}}$, $\mathcal{Q}$ and $\mathcal{Y}$ enter into the cross section together with the spin analysis-power where $\bar{\kappa} = -\kappa$ in CP–invariant theories. Therefore, taking the ratios $\mathcal{P}^2/\mathcal{Q}$ and $\mathcal{P}^2/\mathcal{Y}$, these unknown quantities can be eliminated so that the two ratios reflect unambiguously the properties of the chargino system, not affected by the neutralinos. It is thus possible to study the chargino sector in isolation by measuring the mass of the lightest chargino, the total production cross section and the spin(–spin) correlations. The energy dependence of the two ratios $\mathcal{P}^2/\mathcal{Q}$ and $\mathcal{P}^2/\mathcal{Y}$ is shown in Fig. 4; the same parameters are chosen as in the previous figures. The two ratios are sensitive to the quartic charges at sufficiently large c.m. energies

[4] With event numbers of order 10^5, statistical errors are at the per–mille level.

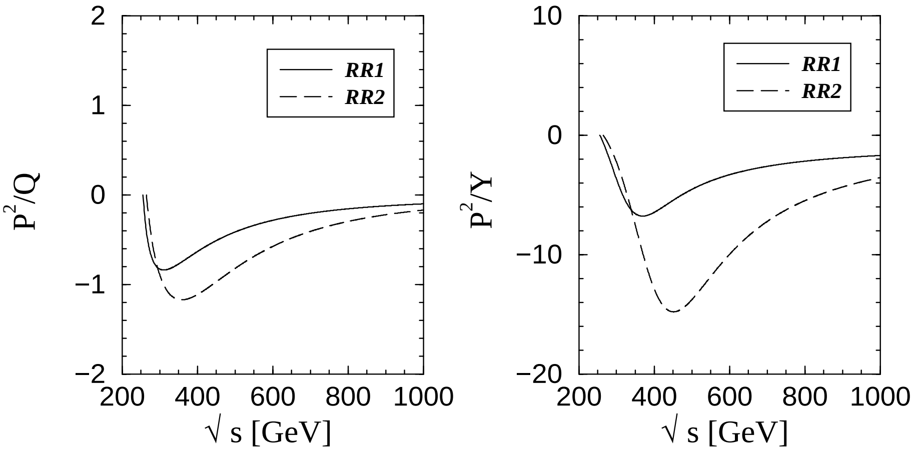

Fig. 4. The energy dependence of the ratios $\mathcal{P}^2/\mathcal{Q}$ and $\mathcal{P}^2/\mathcal{Y}$: solid line for the set $\boldsymbol{RR1}$ [$\tan\beta = 3$, $m_0 = 100$ GeV, $M_{1/2} = 200$ GeV] and dashed line for the set $\boldsymbol{RR2}$ [$\tan\beta = 30$, $m_0 = 160$ GeV, $M_{1/2} = 200$ GeV]

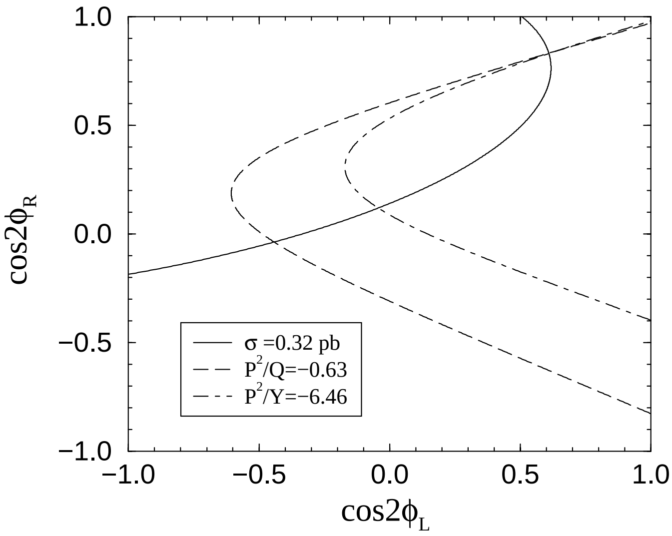

Fig. 5. Contours for the "measured values" of the total cross section (solid line), $\mathcal{P}^2/\mathcal{Q}$ (dashed line), and $\mathcal{P}^2/\mathcal{Y}$ (dot-dashed line) in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane for the set $\boldsymbol{RR1}$ [$\tan\beta = 3$, $m_0 = 100$ GeV, $M_{1/2} = 200$ GeV] at the e^+e^- c.m. energy of 400 GeV

since the charginos are, on the average, unpolarized at the threshold, c.f. (26). Note that $\mathcal{Y}$ vanishes for asymptotic energies so that an optimal energy must be chosen not far above threshold to measure this observable.

The measurement of the cross section at an energy $\sqrt{s}$, and either of the ratios $\mathcal{P}^2/\mathcal{Q}$ or $\mathcal{P}^2/\mathcal{Y}$ can be interpreted as contour lines in the plane $[\cos 2\phi_L, \cos 2\phi_R]$ which intersect at large angles so that a high precision in the resolution can be achieved. A representative example for the determination of $\cos 2\phi_L$ and $\cos 2\phi_R$ is shown in Fig. 5 for the reference point $\boldsymbol{RR1}$. The mass of the light chargino is set to $m_{\tilde{\chi}_1^\pm} = 128$ GeV, and the "measured" cross section, $\mathcal{P}^2/\mathcal{Q}$ and $\mathcal{P}^2/\mathcal{Y}$ are taken to be

$$\sigma\{11\} = 0.32 \text{ pb}\,, \ \mathcal{P}^2/\mathcal{Q} = -0.63\,, \ \mathcal{P}^2/\mathcal{Y} = -6.46 \quad (32)$$

at the e^+e^- c.m. energy $\sqrt{s} = 400$ GeV. The three contour lines meet at a single point $[\cos 2\phi_L, \cos 2\phi_R] = [0.645, 0.844]$.

4.2 The complete chargino system

From the analysis of the complete chargino system $\{\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\tilde{\chi}_2^-, \tilde{\chi}_2^+\tilde{\chi}_2^-\}$, together with the knowledge of the sneutrino mass from sneutrino pair production, the maximal information can be extracted on the basic parameters of the electroweak SU(2) gaugino sector. Moreover, the identity of the $e\tilde{\nu}\tilde{W}$ Yukawa coupling with the $e\nu W$ gauge coupling, which is of fundamental nature in supersymmetric theories, can be tested very accurately. This analysis is the final target of LC experiments which should provide a complete picture of the electroweak gaugino sector with resolution at least at the per-cent level.

The case will be exemplified for the scenario $\boldsymbol{RR1}$ with $\tan\beta = 3$ while the final results will also be presented for $\boldsymbol{RR2}$ with $\tan\beta = 30$. To simplify the picture, without loss of generality, we will not choose separate energies at the maximal values of the cross sections, but instead we will work with a single collider energy $\sqrt{s}$= 800 GeV and an integrated luminosity $\int \mathcal{L} = 1$ ab^{-1}. The polarized cross sections take the following values:

$$\begin{array}{lll} \sigma_R\{11\} = \ \ 1.8\,\text{fb} & \sigma_L\{11\} = 787.7\,\text{fb} & \sigma_T\{11\} = 0.53\,\text{fb} \\ \sigma_R\{12\} = 12.1\,\text{fb} & \sigma_L\{12\} = 106.2\,\text{fb} & \sigma_T\{12\} = 0.53\,\text{fb} \\ \sigma_R\{22\} = 67.1\,\text{fb} & \sigma_L\{22\} = 337.5\,\text{fb} & \sigma_T\{22\} = 1.07\,\text{fb} \end{array} \quad (33)$$

Chargino pair production with right-handed electron beams provides us with the cross sections $\sigma_{R_i} (i = \{11\}, \{12\}, \{22\})$. Due to the absence of the sneutrino exchange diagram, the cross sections can be expressed symmetrically in the mixing parameters

$$\begin{aligned} c_{2L} &= \cos 2\phi_L \\ c_{2R} &= \cos 2\phi_R \end{aligned} \quad (34)$$

as follows:

$$\begin{aligned} \sigma_{R_i} = {} & A_{R_i}\,(c_{2L}^2 + c_{2R}^2) + B_{R_i}\,(c_{2L} + c_{2R}) \\ & + C_{R_i}\, c_{2L} c_{2R} + D_{R_i} \\ & (i = \{11\}, \{12\}, \{22\}) \end{aligned} \quad (35)$$

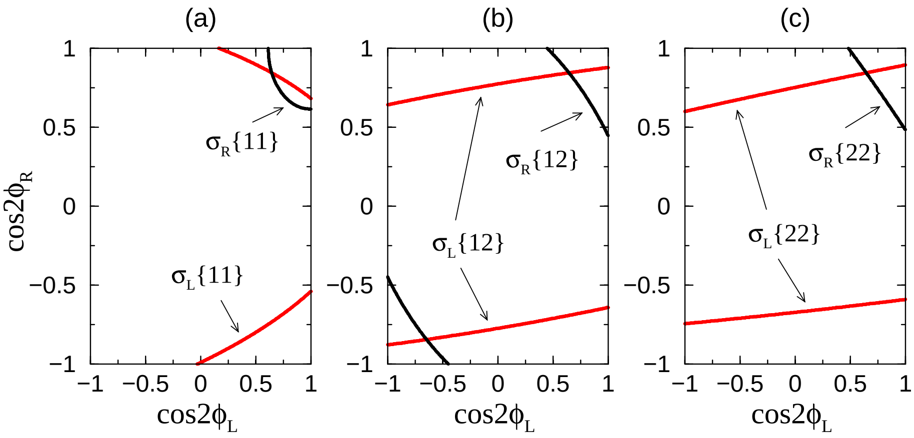

Fig. 6. Contours of the cross sections **a** $\{\sigma_R\{11\}, \sigma_L\{11\}\}$, **b** $\{\sigma_R\{12\}, \sigma_L\{12\}\}$, and **c** $\{\sigma_R\{22\}, \sigma_L\{22\}\}$ in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane for the set ***RR*1** [$\tan\beta = 3$, $m_0 = 100$ GeV, $M_{1/2} = 200$ GeV] at the c.m. energy of 800 GeV

The coefficients A_{R_i}, B_{R_i}, C_{R_i} and D_{R_i} involve only known parameters, the chargino masses and the energy. Depending on whether $A^2_{R_i} \gtrless C^2_{R_i}/4$, the contour lines for $\sigma_R\{11\}, \sigma_R\{12\}, \sigma_R\{22\}$ in the $[c_{2L}, c_{2R}]$ plane (cf. Fig. 6) are either closed ellipses or open hyperbolae[5]. They intersect in two points of the plane which are symmetric under the interchange $c_{2L} \leftrightarrow c_{2R}$; for ***RR*1**: $[c_{2L}, c_{2R}]$ =[0.645,0.844] and interchanged.

While the right–handed cross sections do not involve sneutrino exchange, the cross sections for left–handed electron beams are dominated by the sneutrino contributions unless the sneutrino mass is very large. In general, the three observables σ_{L_i} ($i = \{11\}, \{12\}, \{22\}$) exhibit quite a different dependence on c_{2L} and c_{2R}. In particular, they are not symmetric with respect to c_{2L} and c_{2R} so that the correct solution for $[c_{2L}, c_{2R}]$ can be singled out of the two solutions obtained from the right-handed cross sections (35). As before, the three observables can be expressed as

$$\begin{aligned} \sigma_{L_i} &= A_{L_i}\, c^2_{2L} + A'_{L_i}\, c^2_{2R} + B_{L_i}\, c_{2L} + B'_{L_i}\, c_{2R} \\ &\quad + C_{L_i}\, c_{2L} c_{2R} + D_{L_i} \\ &\quad\quad (i = \{11\}, \{12\}, \{22\}) \end{aligned} \tag{36}$$

The coefficients of the linear and quadratic terms of c_{2L} and c_{2R} depend on known parameters only. The shape of the contour lines is given by the chargino masses and the sneutrino mass, being either elliptic or hyperbolic for $A_{L_i} A'_{L_i} \gtrless C^2_{L_i}/4$, respectively. These asymmetric equations are satisfied *only* by one solution, as shown in Fig. 6. Among the two solutions obtained above from σ_{R_i} only the set $[c_{2L}, c_{2R}] = [0.645, 0.844]$ satisfies (36).

At the same time, the identity between the $e\tilde{\nu}\tilde{W}$ Yukawa coupling and the $e\nu W$ gauge coupling can be tested. Varying the Yukawa coupling freely, the contour lines σ_{L_i} move through the $[c_{2L}, c_{2R}]$ plane. Only for the supersymmetric solutions the curves σ_{L_i} intersect each other and the curves σ_{R_i} in exactly one point. Combining the analyses of σ_{R_i} and σ_{L_i}, the masses, the mixing parameters and the Yukawa coupling can be determined to quite a high precision[6]

$$\begin{aligned} m_{\tilde{\chi}^\pm_1} &= 128 \pm 0.04\,\text{GeV} \qquad \cos 2\phi_L = 0.645 \pm 0.02 \\ m_{\tilde{\chi}^\pm_2} &= 346 \pm 0.25\,\text{GeV} \qquad \cos 2\phi_R = 0.844 \pm 0.005 \\ g_{\tilde{W}}/g_W &= 1 \pm 0.001 \end{aligned} \tag{37}$$

The 1σ statistical errors have been derived for an integrated luminosity of $\int \mathcal{L} = 1\ \text{ab}^{-1}$.

Thus the parameters of the chargino system, masses $m_{\tilde{\chi}^\pm_1}$ and $m_{\tilde{\chi}^\pm_2}$, mixing parameters $\cos 2\phi_L$ and $\cos 2\phi_L$, as well as the Yukawa coupling can be used to extract the fundamental parameters of the underlying supersymmetric theory with high accuracy.

5 The fundamental SUSY parameters

5.1 The $\tilde{\chi}^\pm_1$ base

From the analysis of the $\tilde{\chi}^\pm_1$ states alone, the mixing parameters $\cos 2\phi_L$ and $\cos 2\phi_R$ can be derived unambiguously. This information is sufficient to derive the fundamental gaugino parameters $\{M_2, \mu, \tan\beta\}$ in CP–invariant theories up to at most a discrete two–fold ambiguity.

The solutions can be discussed most transparently by introducing the two triangular quantities

$$p \backslash q = \cot(\phi_R \mp \phi_L) \tag{38}$$

These two quantities can be expressed in terms of the mixing angles:

$$\begin{aligned} p &= \pm \left| \frac{\sin 2\phi_L + \sin 2\phi_R}{\cos 2\phi_L - \cos 2\phi_R} \right| \\ q &= \frac{1}{p}\, \frac{\cos 2\phi_L + \cos 2\phi_R}{\cos 2\phi_L - \cos 2\phi_R} \end{aligned} \tag{39}$$

[5] The cross section $\sigma_R\{12\}$ is always represented by an ellipse.

[6] In contrast to the restricted $\tilde{\chi}^+_1\tilde{\chi}^-_1$ case, it is not necessary to use transversely polarized beams to determine this set of parameters unambiguously. If done so nevertheless, the analysis follows the same steps as discussed above. The additional information will reduce the errors on the fundamental parameters.

Apart from the overall sign ambiguity of the pair (p, q) which can be removed by definition, the set is two–fold ambiguous due to the unfixed relative sign between $\sin 2\phi_L$ and $\sin 2\phi_R$.

From the solutions (p, q) derived above, the SUSY parameters can be determined in the following way:

$$\cos 2\phi_R \gtrless \cos 2\phi_L \ : \ \tan \backslash \cot \beta = \frac{p^2 - q^2 \pm 2\sqrt{\chi^2(p^2+q^2+2-\chi^2)}}{(\sqrt{1+p^2}-\sqrt{1+q^2})^2 - 2\chi^2} \ \Rightarrow \ \tan\beta \gtrless 1 \quad (40)$$

where $\chi^2 = m^2_{\tilde{\chi}^\pm_1}/m^2_W$. The gaugino and higgsino mass parameters are given in terms of p and q by

$$\begin{aligned} M_2 &= \frac{m_W}{\sqrt{2}}\left[(p+q)\sin\beta - (p-q)\cos\beta\right] \\ \mu &= \frac{m_W}{\sqrt{2}}\left[(p-q)\sin\beta - (p+q)\cos\beta\right] \end{aligned} \quad (41)$$

The parameters M_2, μ are uniquely fixed if $\tan\beta$ is chosen properly. Since $\tan\beta$ is invariant under pairwise reflection of the signs in (p, q), the definition $M_2 > 0$ can be exploited to remove this additional ambiguity.

As a result, the fundamental SUSY parameters $\{M_2, \mu, \tan\beta\}$ can be derived from the observables $m_{\tilde{\chi}^\pm_1}$ and $\cos 2\phi_R$, $\cos 2\phi_L$ up to at most a two–fold ambiguity.

5.2 The complete set of the fundamental SUSY parameters

From the set $m_{\tilde{\chi}^\pm_{1,2}}$ and $\cos 2\phi_{L,R}$ of measured observables, the fundamental supersymmetric parameters $\{M_2, |\mu|, \cos\Phi_\mu, \tan\beta\}$ in CP–(non)invariant theories can be determined unambiguously in the following way.

(i) $\boldsymbol{M_2, |\mu|}$: Based on the definition $M_2 > 0$, the gaugino mass parameter M_2 and the modulus of the higgsino mass parameter read as follows:

$$\begin{aligned} M_2 &= \Big[(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m^2_W)/2 \\ &\qquad -(m^2_{\tilde{\chi}^\pm_2} - m^2_{\tilde{\chi}^\pm_1})(\cos 2\phi_L + \cos 2\phi_R)/4\Big]^{1/2} \\ |\mu| &= \Big[(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m^2_W)/2 \\ &\qquad +(m^2_{\tilde{\chi}^\pm_2} - m^2_{\tilde{\chi}^\pm_1})(\cos 2\phi_L + \cos 2\phi_R)/4\Big]^{1/2} \end{aligned} \quad (42)$$

(ii) $\boldsymbol{\cos\Phi_\mu}$: The sign of μ in CP–invariant theories and, more generally, the cosine of the phase of μ in CP–noninvariant theories is determined by the $\tilde{\chi}^\pm_1, \tilde{\chi}^\pm_2$ masses and $\cos 2\phi_{L,R} = c_{2L,R}$

$$\begin{aligned} \cos\Phi_\mu = &\Big[(m^2_{\tilde{\chi}^\pm_2} - m^2_{\tilde{\chi}^\pm_1})^2(2 - c^2_{2L} - c^2_{2R}) \\ &-8m^2_W(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m^2_W)\Big] \\ &\times\Big[16m^4_W - (m^2_{\tilde{\chi}^\pm_2} - m^2_{\tilde{\chi}^\pm_1})^2(c_{2L} - c_{2R})^2\Big]^{-1/2} \\ &\times\Big[4(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m^2_W)^2 \\ &-(m^2_{\tilde{\chi}^\pm_2} - m^2_{\tilde{\chi}^\pm_1})^2(c_{2L} + c_{2R})^2\Big]^{-1/2} . \end{aligned} \quad (43)$$

(iii) $\boldsymbol{\tan\beta}$: The value of $\tan\beta$ is uniquely determined in terms of two chargino masses and two mixing angles:

$$\tan\beta = \sqrt{\frac{4m^2_W - (m^2_{\tilde{\chi}^\pm_2} - m^2_{\tilde{\chi}^\pm_1})(\cos 2\phi_L - \cos 2\phi_R)}{4m^2_W + (m^2_{\tilde{\chi}^\pm_2} - m^2_{\tilde{\chi}^\pm_1})(\cos 2\phi_L - \cos 2\phi_R)}} \quad (44)$$

As a result, the fundamental SUSY parameters $\{M_2, \mu, \tan\beta\}$ in CP–invariant theories, and $\{M_2, |\mu|, \cos\Phi_\mu, \tan\beta\}$ in CP–noninvariant theories, can be extracted unambiguously from the observables $m_{\tilde{\chi}^\pm_{1,2}}$, $\cos 2\phi_R$, and $\cos 2\phi_L$. The final ambiguity in $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$ in CP–noninvariant theories must be resolved by measuring observables related to the normal $\tilde{\chi}^-_1$ or/and $\tilde{\chi}^+_2$ polarization in non–diagonal $\tilde{\chi}^-_1\tilde{\chi}^+_2$ chargino–pair production [13].

For illustration, the accuracy which can be expected in such an analysis, is shown for both CP–invariant reference points ***RR*1** and ***RR*2** in Table 2. If $\tan\beta$ is large, this parameter is difficult to extract from the chargino sector. Since the chargino observables depend only on $\cos 2\beta$, the dependence on β is flat for $2\beta \to \pi$ so that (44) is not very useful to derive the value of $\tan\beta$ due to error propagation. A significant lower bound can be derived nevertheless in any case.

5.3 Two–state completeness relations

The two–state mixing of charginos leads to sum rules for the chargino couplings. They can be formulated in terms of the squares of the bilinear charges, *i.e.* the elements of the quartic charges. This follows from the observation that the mixing matrix is built up by trigonometric functions among which many relations are valid. From evaluating these sum rules experimentally, it can be concluded whether the two–chargino system $\{\tilde{\chi}^\pm_1, \tilde{\chi}^\pm_2\}$ forms a closed system, or whether additional states, at high mass scales, mix in.

The following general sum rules can be derived for the two–state charginos system at tree level:

$$\sum_{i,j=1,2} |Q_{\alpha\beta}|^2\{ij\} = 2\,(|D_\alpha|^2 + |F_\alpha|^2) \qquad (\alpha\beta) = (LL, RL, RR) \quad (45)$$

The right–hand side is independent of any supersymmetric parameters, and it depends only on the electroweak parameters $\sin^2\theta_W, m_Z$ and on the energy, cf. (13). Asymptotically, the initial energy dependence and the m_Z dependence drop out. The corresponding sum rule for the mixed left–right (LR) combination,

$$\sum_{i,j=1,2} |Q_{LR}|^2\{ij\} = 2(|D'_L|^2 + |F'_L|^2) \quad (46)$$

Table 2. Estimate of the accuracy with which the parameters M_2, μ, $\tan\beta$ can be determined, including $\mathrm{sgn}(\mu)$, from chargino masses and production cross sections; errors at the 1σ level are statistical only

	***RR*1**		***RR*2**	
	theor. value	fit value	theor. value	fit value
M_2	152 GeV	152 ± 1.75 GeV	150 GeV	150 ± 1.2 GeV
μ	316 GeV	316 ± 0.87 GeV	263 GeV	263 ± 0.7 GeV
$\tan\beta$	3	3 ± 0.69	30	> 20.2

involves the sneutrino mass and Yukawa coupling.

The validity of these sum rules is reflected in both the quartic charges and the production cross sections. However, due to mass effects and the t–channel sneutrino exchange, it is not straightforward to derive the sum rules for the quartic charges and the production cross sections in practice. Only *asymptotically* at high energies the sum rules (45) for the charges can be transformed directly into sum rules for the associated cross sections:

$$\sum_{i,j=1,2} \sigma_{L,R}\{ij\} \simeq \frac{16\pi\alpha^2}{3s}\left(|D_{L,R}|^2 + |F_{L,R}|^2\right) \tag{47}$$

For non–asymptotic energies the fact that all the physical observables are bilinear in $\cos 2\phi_L$ and $\cos 2\phi_R$, enables us nevertheless to relate the cross sections with the set of the six variables $\vec{z} = \{1, c_{2L}, c_{2R}, c_{2L}^2, c_{2R}^2, c_{2L}c_{2R}\}$. For the sake of simplicity we restrict ourselves to the left and right–handed cross sections. We introduce the generic notation $\vec{\sigma}$ for the six cross sections $\sigma_R\{ij\}$ and $\sigma_L\{ij\}$:

$$\vec{\sigma} = \Big\{\sigma_R\{11\}, \sigma_R\{12\}, \sigma_R\{22\}, \sigma_L\{11\}, \sigma_L\{12\}, \sigma_L\{22\}\Big\} \tag{48}$$

Each cross section can be decomposed in terms of c_{2L} and c_{2R} by noting that

$$\sigma_i = \sum_{j=1}^{6} f_{ij}[m^2_{\tilde{\chi}^\pm_{1,2}}, m^2_{\tilde{\nu}}]\, z_j \tag{49}$$

The matrix elements f_{ij} can easily be derived from Table 1 together with (13-16). Since the observables σ_R do not involve sneutrino contributions, the corresponding functions f_{ij} do not depend on the sneutrino mass. The $6{\times}6$ matrix f_{ij} relates the six left/right–handed cross sections and the six variables z_i. Inverting the matrix gives the expressions for the variables z_i in terms of the observables. Since the variables z_i are not independent, we obtain several non–trivial relations among the observables of the chargino sector:

$$z_1 = 1 \quad : \quad f^{-1}_{1j}\sigma_j = 1 \tag{50}$$

$$z_4 = z_2^2 \quad : \quad f^{-1}_{4j}\sigma_j = \left[f^{-1}_{2j}\sigma_j\right]^2 \tag{51}$$

$$z_5 = z_3^2 \quad : \quad f^{-1}_{5j}\sigma_j = \left[f^{-1}_{3j}\sigma_j\right]^2 \tag{52}$$

$$z_6 = z_2 z_3 : \quad f^{-1}_{6j}\sigma_j = f^{-1}_{2j} f^{-1}_{3k}\sigma_j\sigma_k \tag{53}$$

where summing over repeated indices is understood. The failure of saturating any of these sum rules by the measured cross sections would signal that the chargino two–state $\{\tilde{\chi}^\pm_1, \tilde{\chi}^\pm_2\}$ system is not complete and additional states mix in.

6 Conclusions

We have analyzed in this report how the parameters of the chargino system, the chargino masses $m_{\tilde{\chi}^\pm_{1,2}}$ and the size of the wino and higgsino components in the chargino wave–functions, parameterized by the two mixing angles ϕ_L and ϕ_R, can be extracted from pair production of the chargino states in e^+e^- annihilation. Three production cross sections $\tilde{\chi}^+_1\tilde{\chi}^-_1$, $\tilde{\chi}^+_1\tilde{\chi}^-_2$, $\tilde{\chi}^+_2\tilde{\chi}^-_2$, for left– and right–handedly polarized electrons give rise to six independent observables. The method is independent of the chargino decay properties, *i.e.* the analysis is not affected by the structure of the neutralino sector which is generally very complex in supersymmetric theories while the chargino sector remains generally isomorphic to the minimal form of the MSSM.

The measured chargino masses $m_{\tilde{\chi}^\pm_{1,2}}$ and the two mixing angles ϕ_L and ϕ_R allow us to extract the fundamental SUSY parameters $\{M_2, \mu, \tan\beta\}$ in CP–invariant theories unambiguously; in CP–noninvariant theories the modulus of μ and the cosine of the phase can be determined, leaving us with just a discrete two–fold ambiguity $\phi_\mu \leftrightarrow 2\pi - \phi_\mu$ which can be resolved by measuring the sign of observables associated with the normal $\tilde{\chi}^\pm_{1,2}$ polarizations.

Sum rules for the production cross sections can be used at high energies to check whether the two–state chargino system is a closed system or whether additional states mix in from potentially high scales.

To summarize, the measurement of the processes $e^+e^- \to \tilde{\chi}^+_i\tilde{\chi}^-_j$ $[i,j = 1,2]$ carried out with polarized beams, leads to a complete analysis of the basic SUSY parameters $\{M_2, \mu, \tan\beta\}$ in the chargino sector. Since the analysis can be performed with high precision, this set provides a solid platform for extrapolations to scales eventually near the Planck scale where the fundamental supersymmetric theory may be defined.

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References

1. For reviews, see H. Nilles, Phys. Rep. **110** (1984) 1; H.E. Haber, G.L. Kane, Phys. Rep. **117** (1985) 75
2. J. Ellis, J. Hagelin, D. Nanopoulos, M. Srednicki, Phys. Lett. **127B**(1983) 233; V. Barger, R.W. Robinett, W.Y. Keung, R.J.N. Phillips, Phys. Lett. B **131** (1993) 372; D. Dicus, S. Nandi, W. Repko, X. Tata, Phys. Rev. Lett **51** (1983) 1030; S. Dawson, E. Eichten, C. Quigg, Phys. Rev. D **31** (1985) 1581; A. Bartl, H. Fraas, W. Majerotto, Z. Phys. C **30** (1986) 441
3. T. Tsukamoto, K. Fujii, H. Murayama, M. Yamaguchi, Y. Okada, Phys. Rev. D **51** (1995) 3153; J.L. Feng, M.E. Peskin, H. Murayama, X. Tata, Phys. Rev. D **52** (1995) 1418
4. E. Accomando et al., Phys. Rep. **299** (1998) 1, and LC CDR Report DESY/ECFA 97–048/182
5. S.Y. Choi, A. Djouadi, H. Dreiner, J. Kalinowski, P.M. Zerwas, Eur. Phys. J. C **7** (1999) 123
6. S.Y. Choi, A. Djouadi, H.S. Song, P.M. Zerwas, Eur. Phys. J. C **8** (1999) 669
7. S.Y. Choi, M. Guchait, J. Kalinowski, P.M. Zerwas, hep-ph/0001175
8. G. Moortgat–Pick, H. Fraas, A. Bartl, W. Majerotto, Eur. Phys. J. C **7** (1999) 113; G. Moortgat–Pick, H. Fraas, Acta Phys. Polon. B **30** (1999) 1999
9. V. Lafage et al., KEK–Report KEK–CP–078 (Oct. 1998), [hep–ph/9810504]
10. V. Barger, T. Han, T.-J. Li, T. Plehn, Madison Report MADPH–99–1128 [hep–ph/9907425]
11. G. Moortgat–Pick, A. Bartl, H. Fraas, W. Majerotto, DESY Report 00–003
12. W. Bernreuther, O. Nachtmann, P. Overmann, T. Schröder, Nucl. Phys. B **388** (1992) 53
13. Y. Kizukuri, N. Oshimo, Proceedings of the Workshop on e^+e^- Collisions at 500 GeV: The Physics Potential, Munich-Annecy-Hamburg 1993, DESY 93-123C, ed. P. Zerwas; T. Ibrahim, P. Nath, Phys. Rev. D **57** (1998) 478; M. Brhlik, G.L. Kane, Phys. Lett. B **437** (1998) 437; M. Brhlik, G.J. Good, G.L. Kane, Phys. Rev. D **59** (1999) 115004
14. For recent developments, see J.-L. Kneur, G. Moultaka, Phys. Rev. D **59** (1999) 015005; G. Moortgat–Pick, A. Bartl, H. Fraas, W. Majerotto, DESY Report 00–002
15. M.A. Diaz, S.F. King, D.A. Ross, Nucl. Phys. B **529** (1998) 23; M.A. Diaz, hep–ph/9910445
16. L.M. Sehgal, P.M. Zerwas, Nucl. Phys. B **183** (1981) 417
17. J.H. Kühn, F. Wagner, Nucl. Phys. B **236** (1994) 16
18. G. Moortgat-Pick, H. Fraas, Acta Phys. Polon. B **30** (1999) 1999; G. Moortgat–Pick, H. Fraas, A. Bartl, W. Majerotto, Acta. Phys. Polon. **29B** (1998) 1497; Eur. Phys. J. C **9** (1999) 521; ibid. C **9** (1999) 549 (Erratum); S.Y. Choi, H.S. Song, W.Y. Song, hep–ph/9907474
19. S. Ambrosanio, G. Blair, P.M. Zerwas, in DESY/ECFA 1998/99 LC Workshop. For more details, see the web site http://www.desy.de/conferences/ecfa–desy–lc98.html
20. G. Blair, H.U. Martyn, hep–ph/9910416